USE OF STATISTICS OF EXTREMES TO DETECT TRENDS IN HURRICANE STATISTICS

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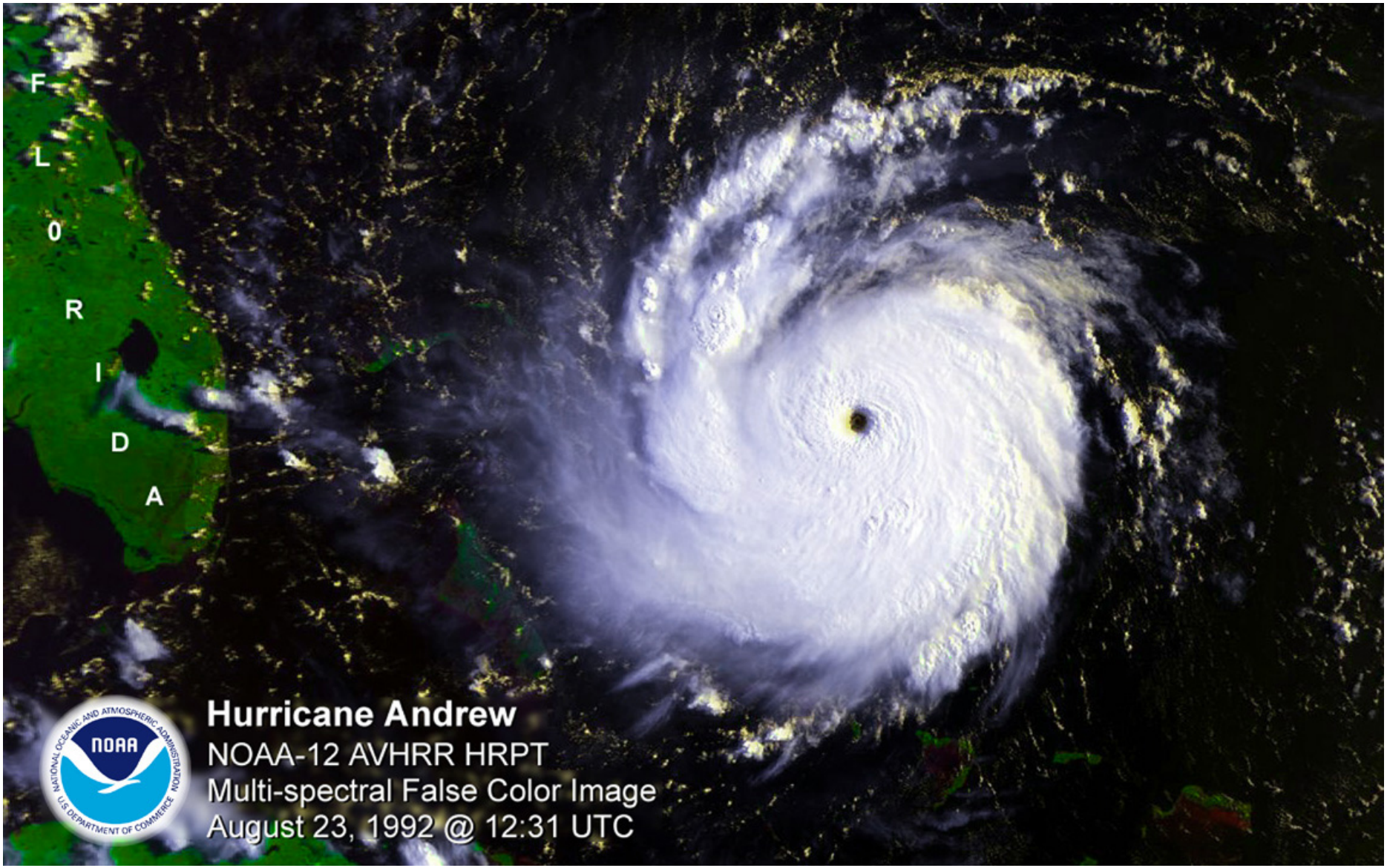
Quote

John Tukey (1915 – 2000):

"*As I am sure almost every geophysicist knows, distributions of actual errors and fluctuations have much more straggling extreme values than would correspond to the magic bell-shaped distribution of Gauss and Laplace.*"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Hurricane Andrew
NOAA-12 AVHRR HRPT
Multi-spectral False Color Image
August 23, 1992 @ 12:31 UTC

Outline

(1) Economic Damage from Hurricanes

(2) Stochastic Model for Damage

(3) Covariates

(4) Opportunities for Extreme Value Theory

(5) Resources

(1) Economic Damage from Hurricanes

- **Data**

-- Previous analysis for time period 1925 – 1995

Katz (2002) fit "random sum" model to hurricane damage data,

Found El Niño signal, but no trends

-- Extended data set for time period 1900 – 2005

Pielke et al. (2007, in press): Paper and data set posted at

`sciencepolicy.colorado.edu/`
`publications/special/normalized_hurricane_damages.html`

- **“Normalized” Data**

-- Adjusted for inflation & changes in societal vulnerability (US$ 2005)

Increased population along coast

Increased wealth along coast

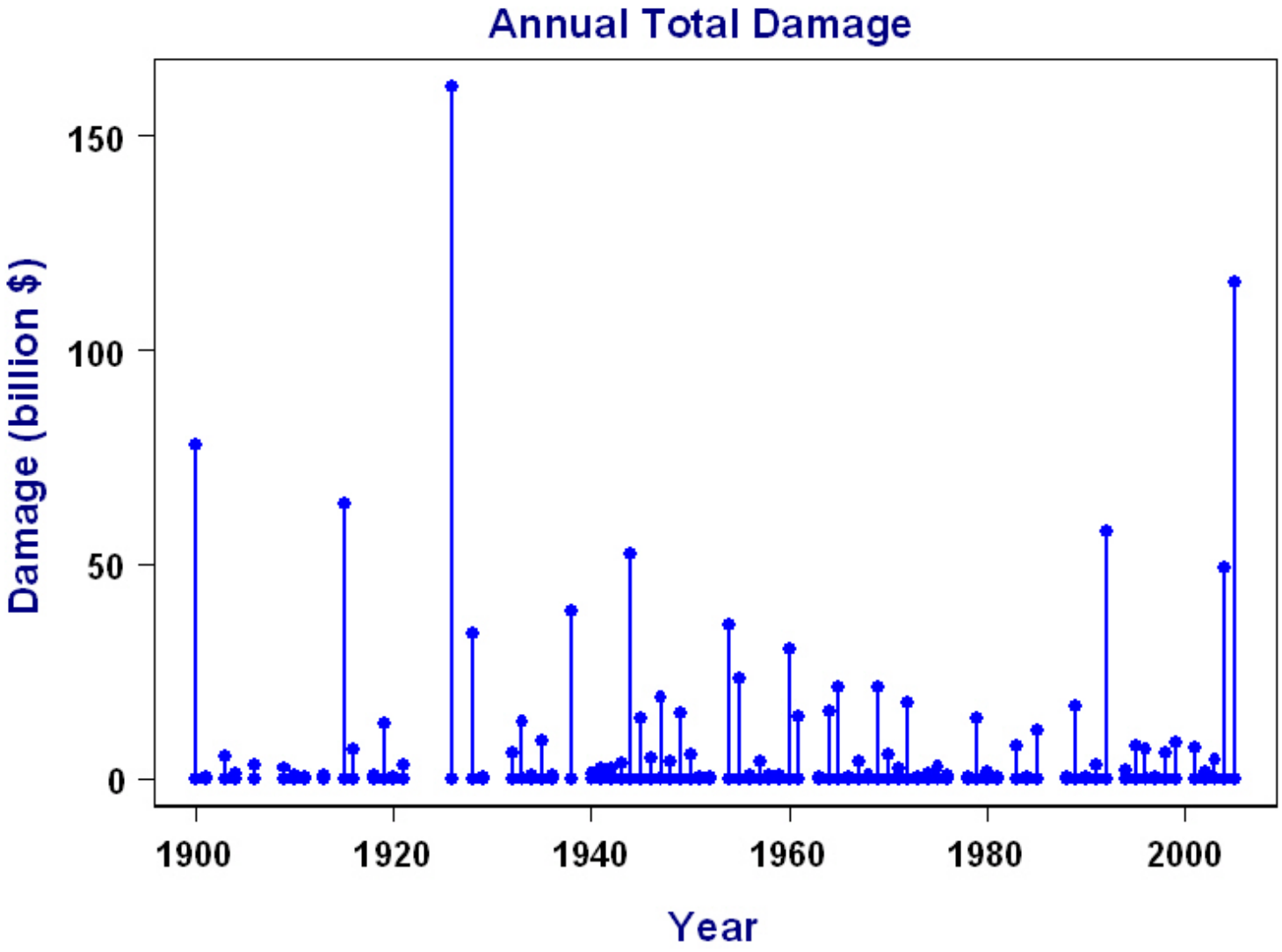
-- Limitations

Uncertainties in adjustment process (not quantified)

Bias against low damage events early in record

(Exclude events with damage < $0.1 billion)

Ignores any interaction (Climate – Society)


Annual Total Damage
Damage (billion $)
150
100
50
0
1900
1920
1940
1960
1980
2000
Year

(2) Stochastic Model for Damage

- **Random Sum Model**

-- **Embrechts et al. (1997):**

"Bread and butter of insurance mathematics"

- **Number of Events ("Law of Small Numbers")**

-- **Poisson distribution (Trend? Covariates?)**

- **Damage from Individual Storms**

-- **Lognormal distribution (Trend? Covariates?)**

-- **Generalized Pareto (GP) distribution for upper tail**

- **Statistics of Random Sums**

-- **Notation**

$N(t)$ number of events in tth yr

X_k damage from kth event in tth yr (independent & identically dist.)

Total damage in tth yr:

$$S(t) = X_1 + X_2 + \cdots + X_{N(t)}, \quad N(t) > 0$$

-- **Mean of total annual damage**

$$E[S(t)] = E[N(t)]\, E(X_k)$$

-- Variance of total annual damage

$$\mathrm{Var}[S(t)] = E[N(t)]\,\mathrm{Var}(X_k) + \mathrm{Var}[N(t)]\,[E(X_k)]^2$$

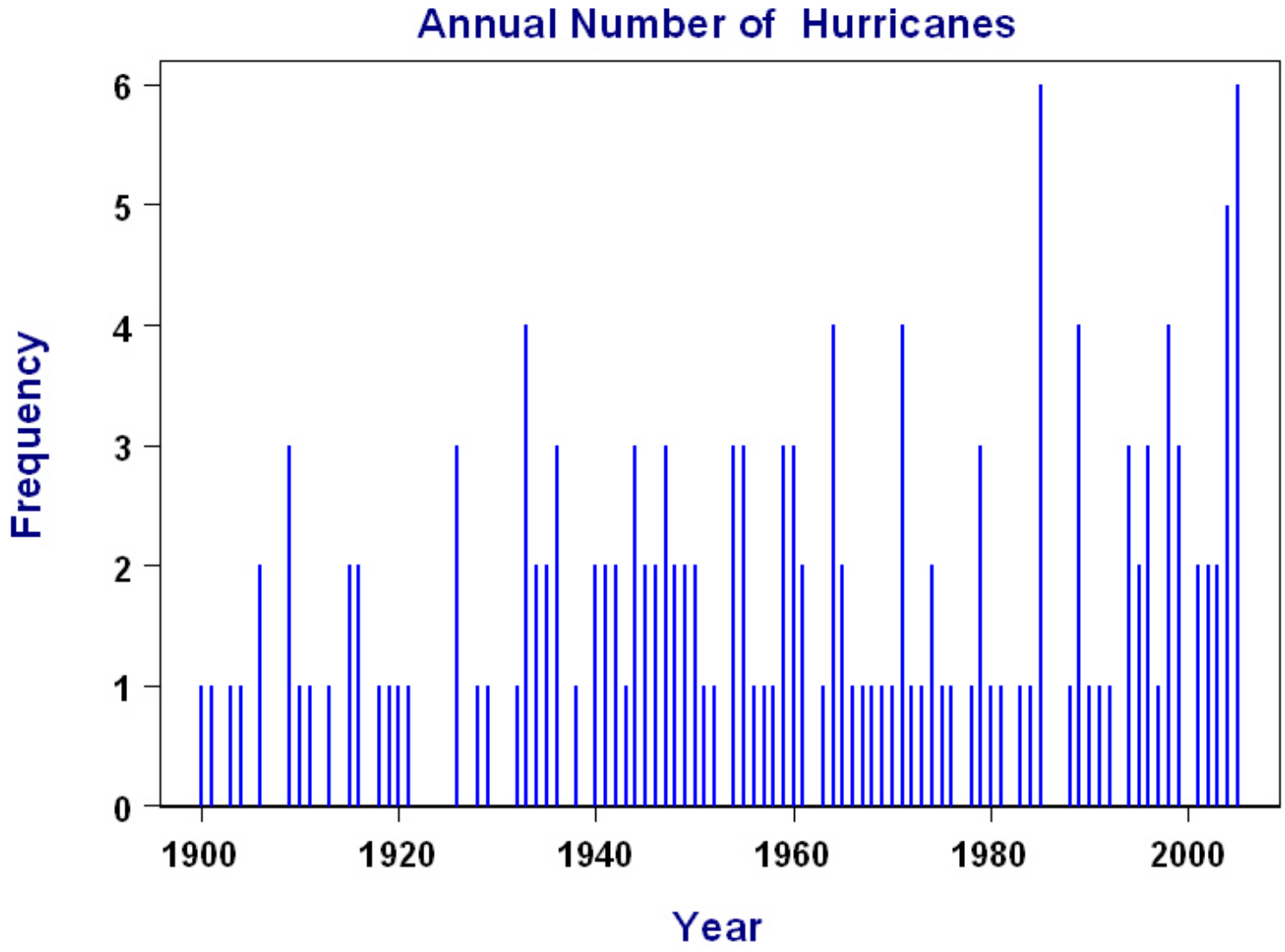
-- Distribution of $S(t)$

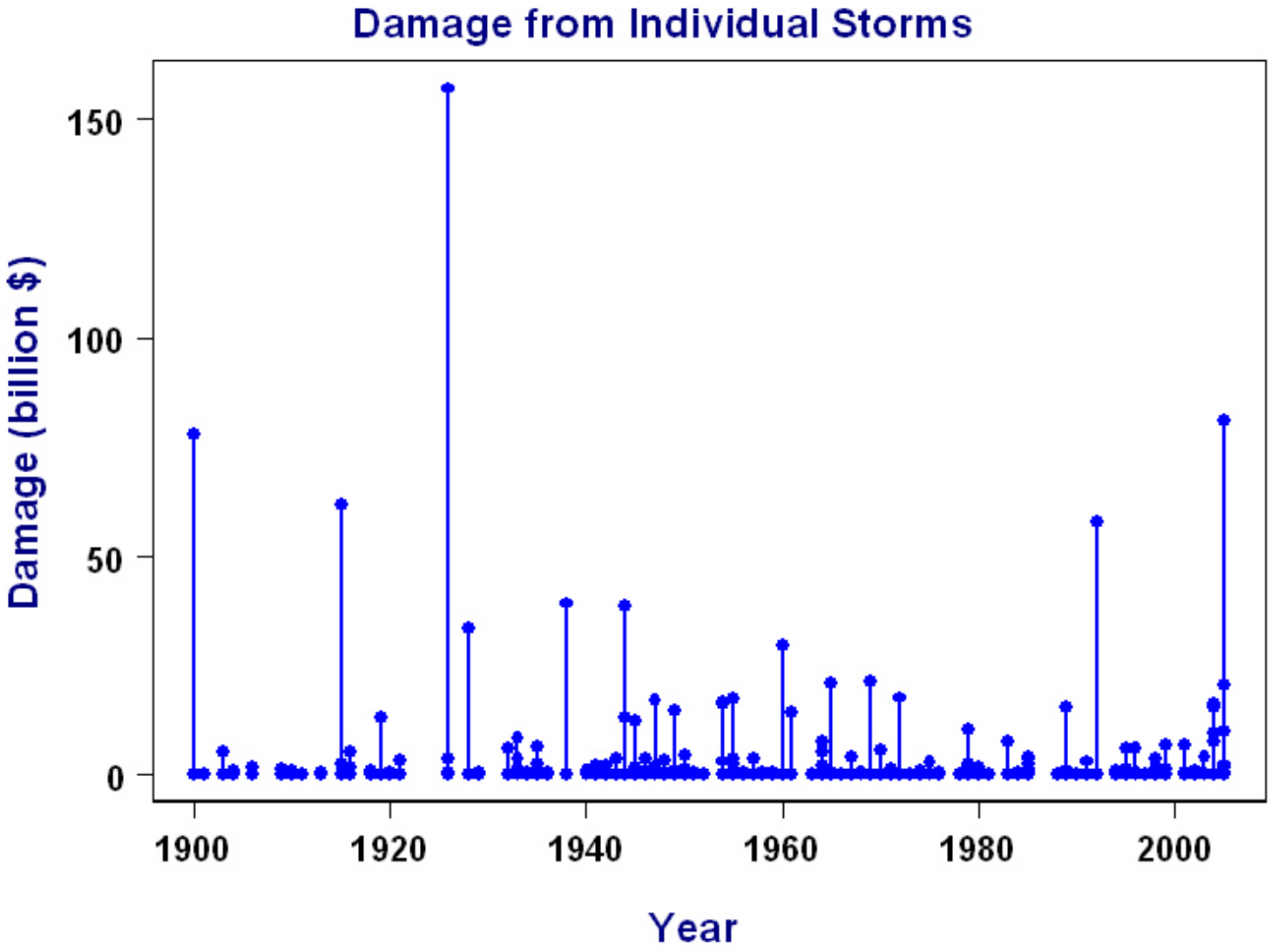
Exact distribution complex

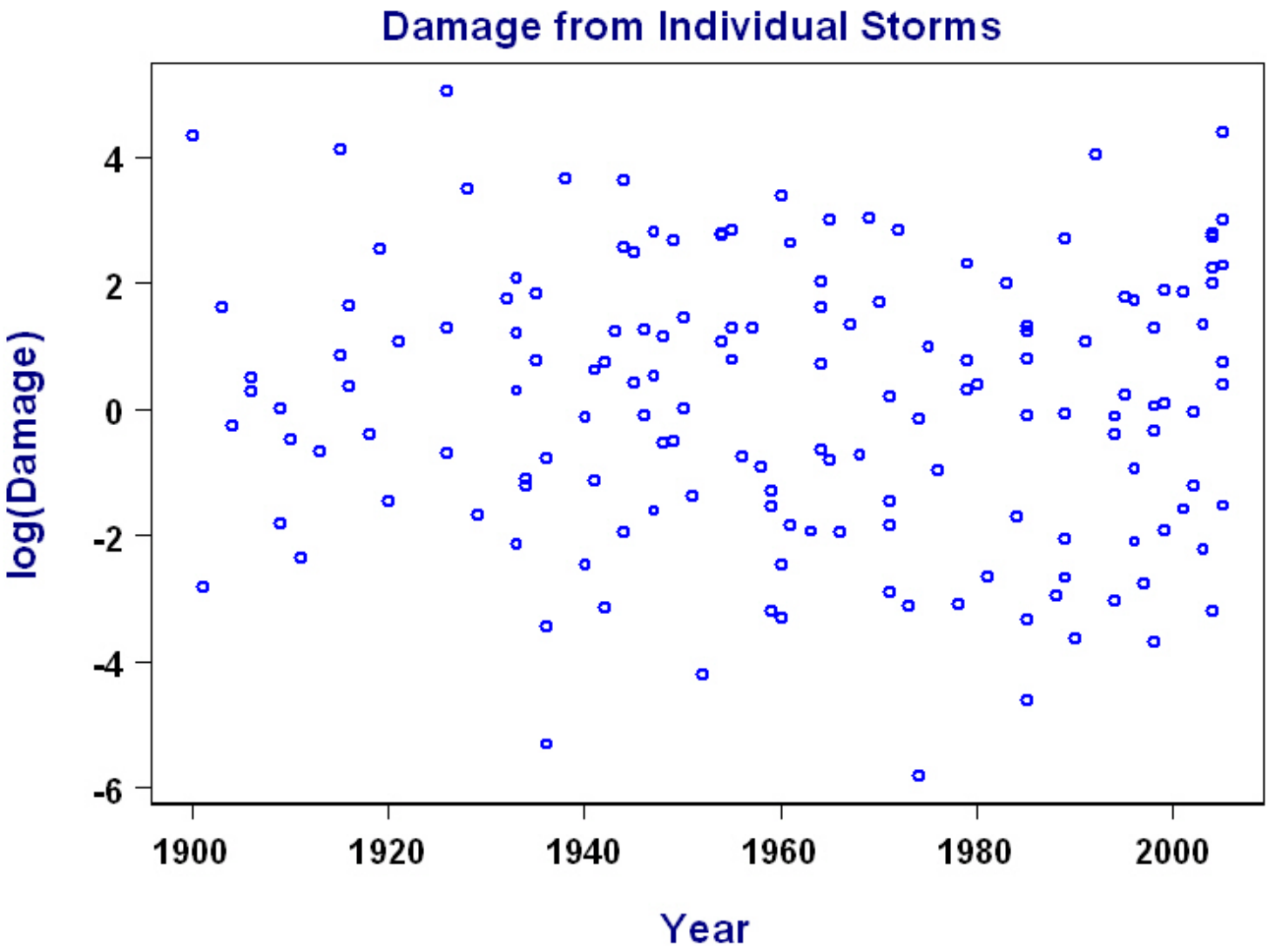
(e. g., for $N(t)$ Poisson & X_k lognormal)

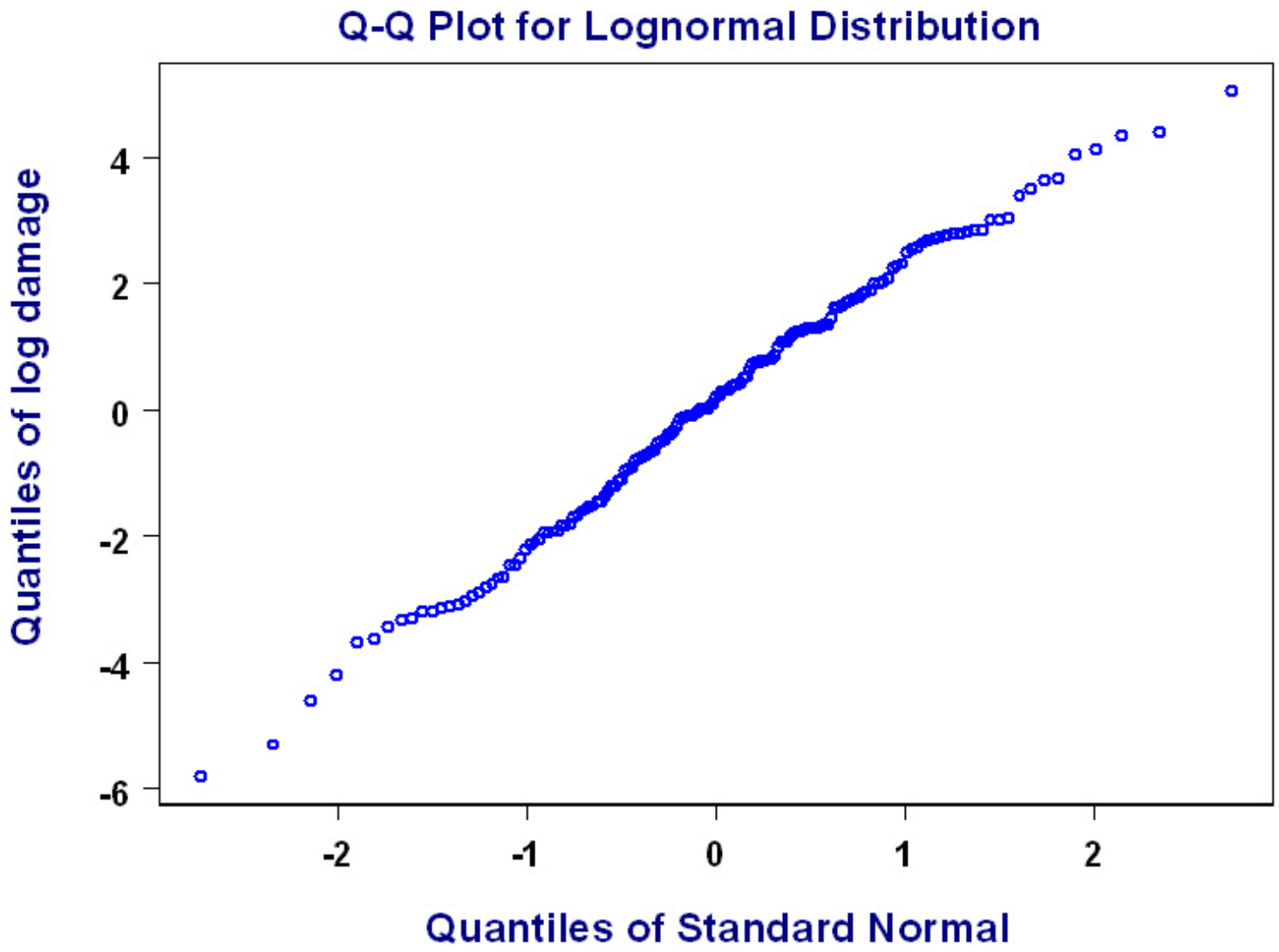
Upper tail behavior of $S(t)$ governed by upper tail of distribution of X_k

Exact result for distribution of maximum of *random* number of random variables when $N(t)$ Poisson (i. e., highest damage from individual storm in given year)


Annual Number of Hurricanes
Frequency
0
1
2
3
4
5
6
1900
1920
1940
1960
1980
2000
Year


Damage from Individual Storms
Damage (billion $)
150
100
50
0
1900
1920
1940
1960
1980
2000
Year


Damage from Individual Storms
log(Damage)
4
2
0
-2
-4
-6
1900
1920
1940
1960
1980
2000
Year


Q-Q Plot for Lognormal Distribution
Quantiles of log damage
4
2
0
-2
-4
-6
-2
-1
0
1
2
Quantiles of Standard Normal

- **Heavy Upper Tail**

-- **"Excess" in damage over $10 billion (26 storms)**

-- **Estimated shape parameter of Generalized Pareto distribution ≈ 0.44**

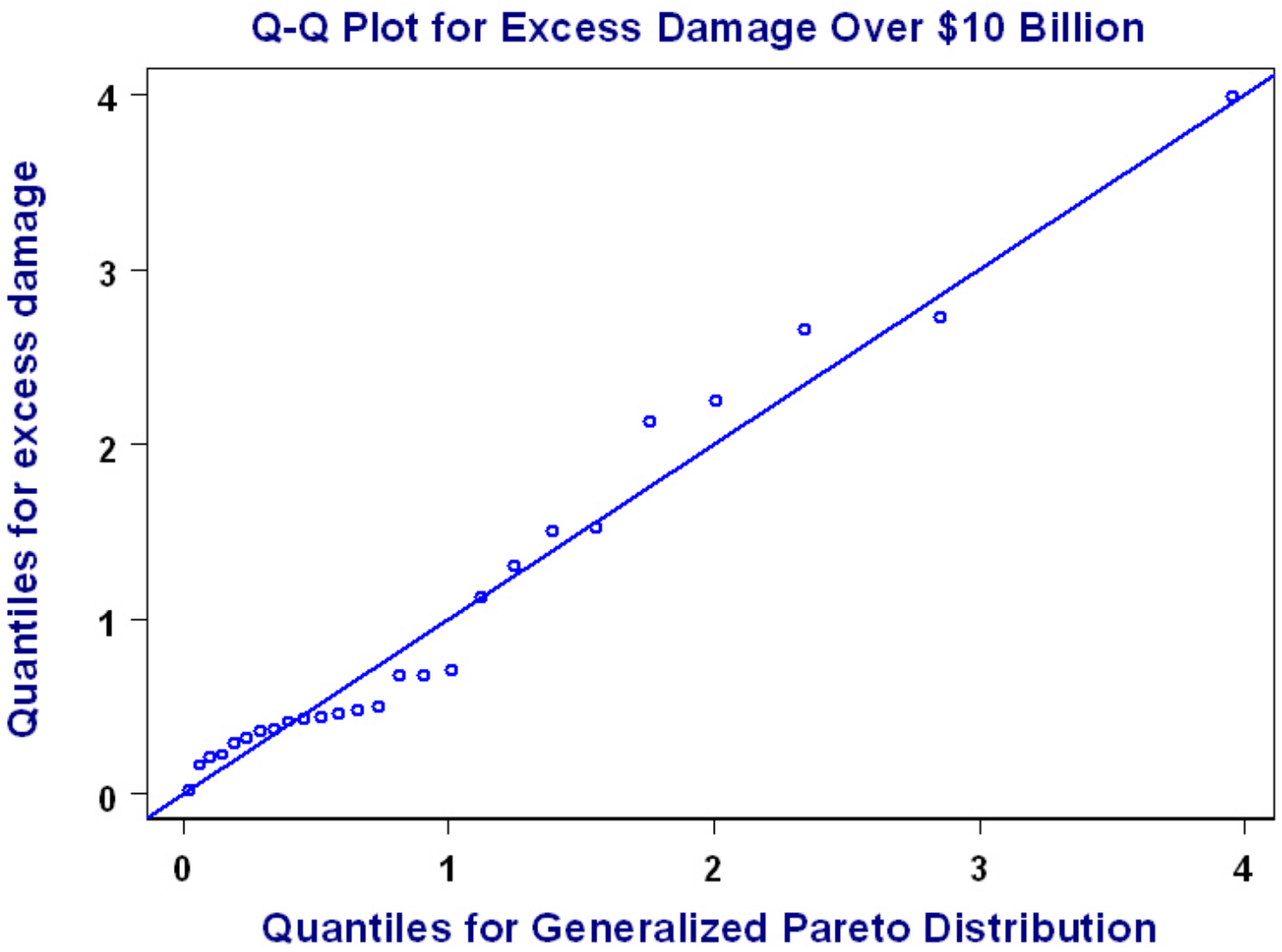
- **Origin of Heavy Tail**

-- **Underlying geophysical phenomenon?**

-- **Inherent feature of distribution of income or wealth?**
(Recall origin of Pareto distribution)

-- **Chance Mechanisms**

Mixture of light-tailed distributions can induce heavy-tailed distribution
(e. g., exponential to Pareto)


Q-Q Plot for Excess Damage Over $10 Billion
Quantiles for excess damage
0
1
2
3
4
0
1
2
3
4
Quantiles for Generalized Pareto Distribution

(3) Covariates

- El Niño Phenomenon

-- Statistical characteristics (“quasi-periodic”)

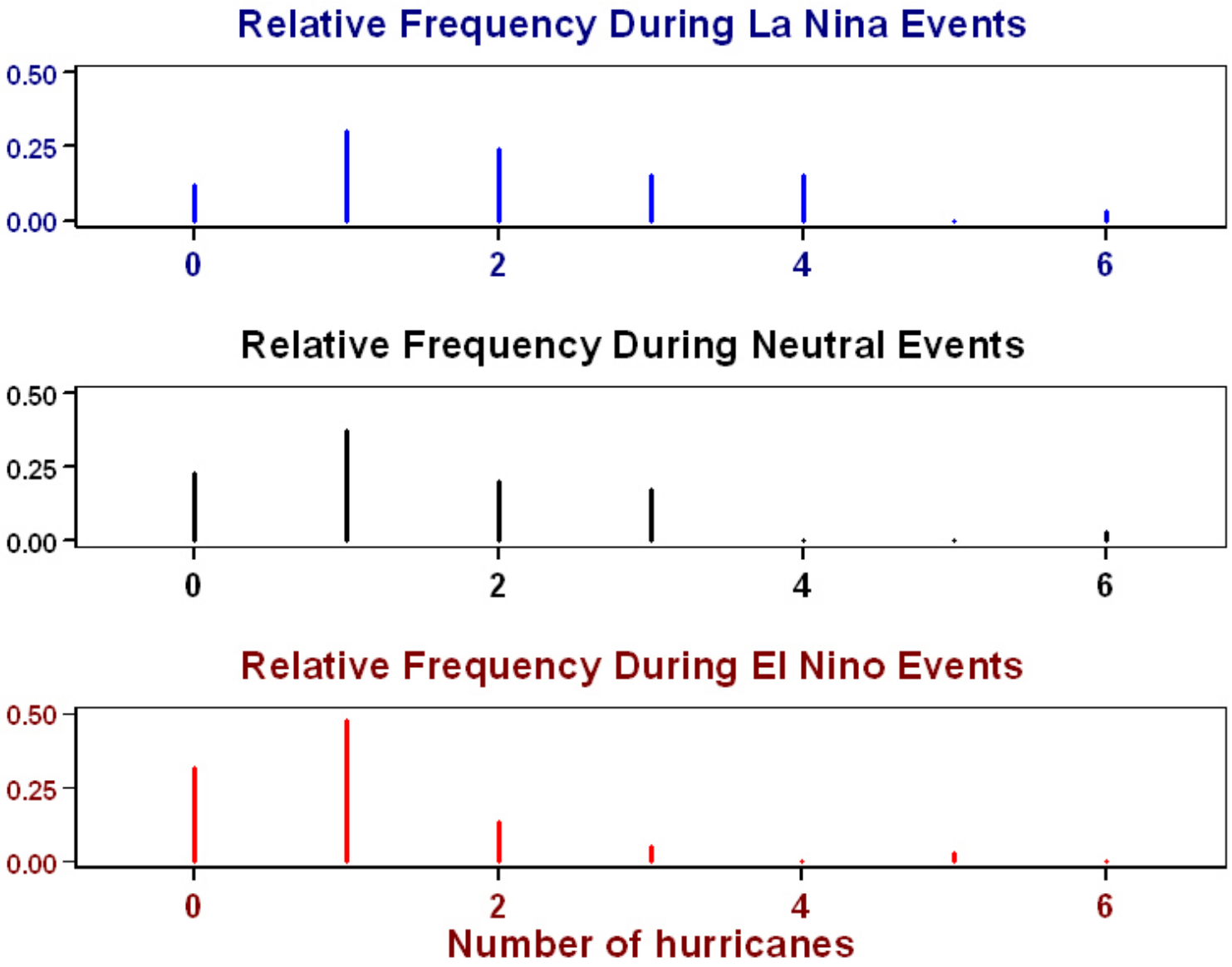
-- Teleconnections (interannual variability)

- Connections to Hurricane Statistics

-- Hurricane frequency (Basis for seasonal forecasts)

-- Hurricane intensity?

-- Hurricane path (North Atlantic Oscillation, Bermuda High)


Relative Frequency During La Nina Events
0.50
0.25
0.00
0
2
4
6
Relative Frequency During Neutral Events
0.50
0.25
0.00
0
2
4
6
Relative Frequency During El Nino Events
0.50
0.25
0.00
0
2
4
6
Number of hurricanes

- **El Niño Effect on Frequency of Hurricanes**

-- **Denote El Niño state in year t by $Z(t) = -1, 0, 1$**

-- **Mean number of events**

La Niña year $Z(t) = -1$: 2.03 events per year

Neutral year $Z(t) = 0$: 1.46 events per year

El Niño year $Z(t) = 1$: 1.03 events per year

- **Model Combining El Niño Effect & Trend**

-- **Rate parameter $\lambda(t)$**

Conditional mean of Poisson distribution for year t given $Z(t)$:

$$\log \lambda(t) = \lambda_0 + \lambda_1 Z(t) + \lambda_2 t, \quad t = 1900, 1901, \ldots, 2005$$

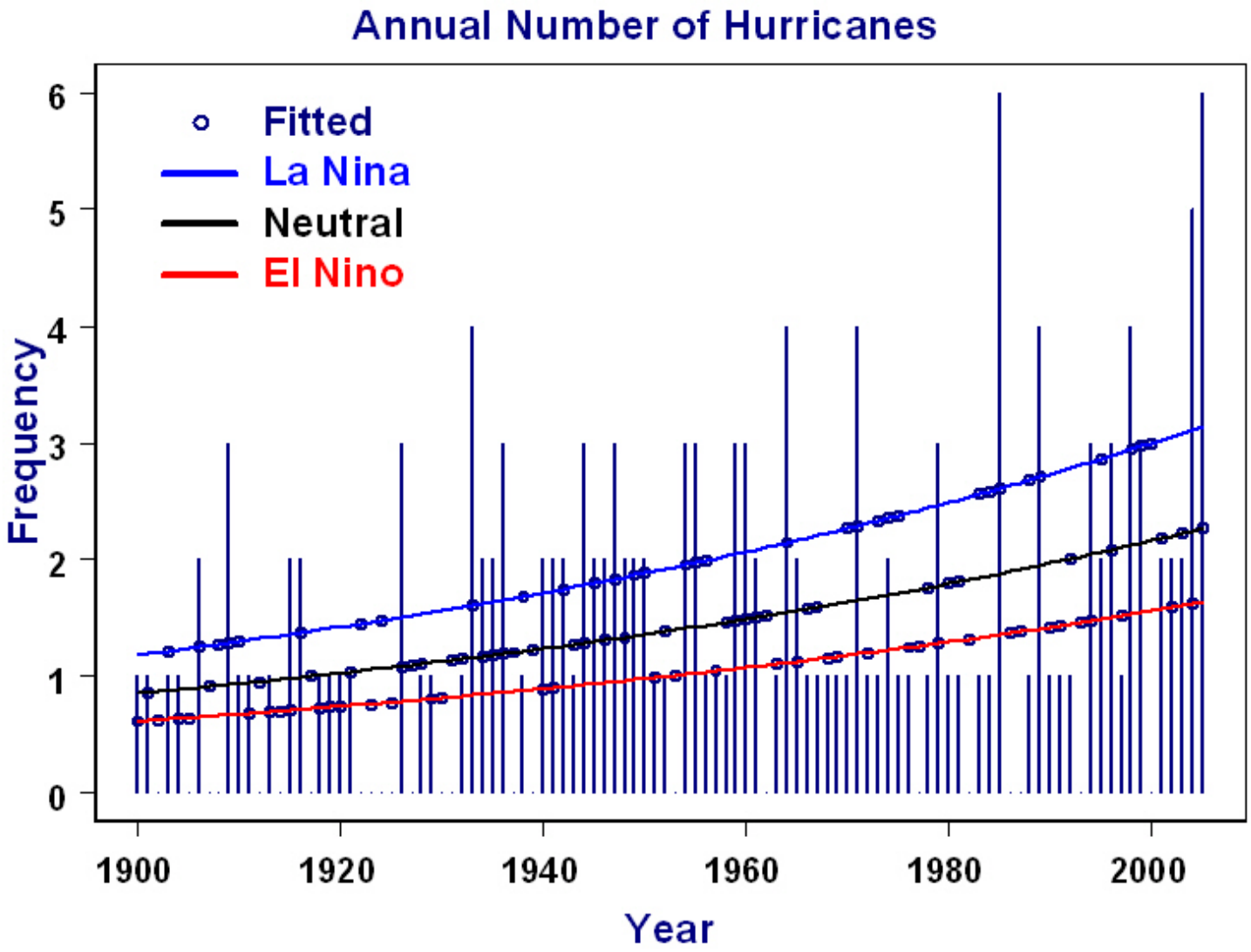
-- Fit model using generalized linear model (GLM)

Likelihood ratio test for $\lambda_1 = 0$: P-value < 0.001

Likelihood ratio test for $\lambda_2 = 0$: P-value < 0.001

-- Form of El Niño index

Only small improvement in fit if use continuous sea surface temperature index for El Niño (instead of three discrete categories)


Annual Number of Hurricanes
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El Nino
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Year

- **Individual Storm Damage**

-- **El Niño effect**

No effect detected

(Katz 2002: Borderline statistical significance)

-- **Trend in mean of lognormal distribution**

(or scale parameter of Generalized Pareto distribution)

No trend detected:

Conflict with theory of increased intensity under global warming

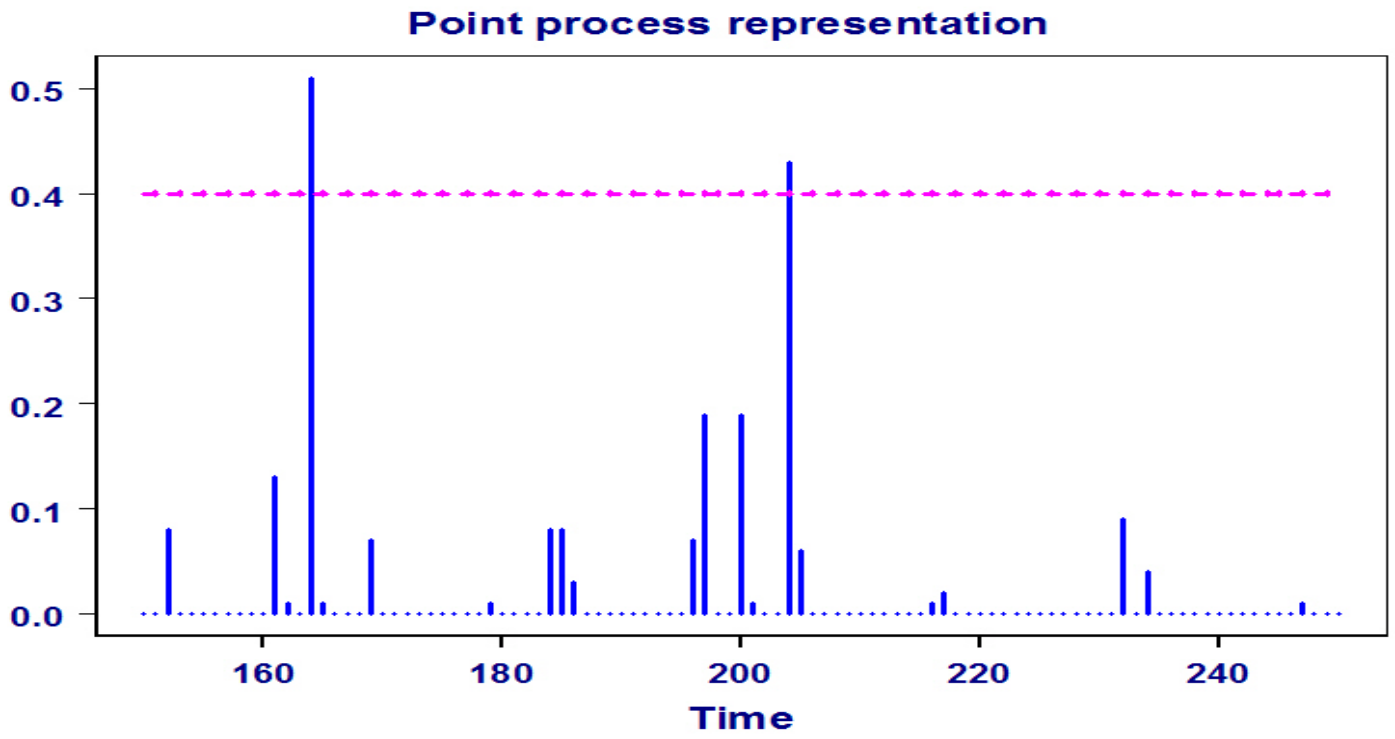
Conflict with trend in observed intensity (much larger sample)

Reflects lack of statistical power?

(4) Opportunities for Extreme Value Theory

- **Point Process Approach to Extremes**

-- **Incorporate seasonality, trends in seasonality, etc.**



- **Maximum Hurricane Intensity**

-- **Emanuel (2000)**

Fit bounded distribution to maximum wind speed

-- **Estimation of upper bound**
(Natural application of extreme value theory)

-- **Increasing trend in upper bound with global warming**
(Introduce trend as covariate)

-- **Issue of penultimate approximations in extreme value theory**
(Apparent finite upper bound, when ultimate bound is infinite)

- **Multidecadal Variation**

-- **Alternative explanation for long-term trend for hurricane statistics**

-- **Concept not defined in rigorous statistical manner**

Conditional nonstationarity vs. unconditional nonstationarity

-- **Hidden Markov model**

Natural candidate for “regimes”

Recently proposed as model for North Atlantic Oscillation

(5) Resources

- **Statistics of Weather and Climate Extremes**

-- `www.isse.ucar.edu/extremevalues/extreme.html`

-- **Web page focusing on application of statistics of extremes to weather and climate (maintained by me)**

- **Extremes Toolkit (`extRemes`)**

-- `www.isse.ucar.edu/extremevalues/evtk.html`

-- **Open source software in `R` with GUIs (developed by Eric Gilleland)**