SEAMOCS Workshop

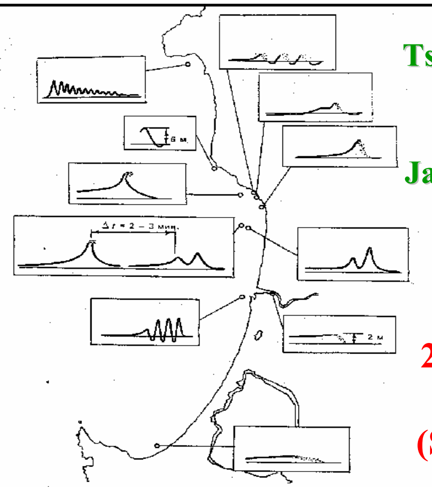
Runup of the sea waves on a beach

Ira Didenkulova

Institute of Applied Physics, Russian Academy of Sciences, Nizhny Novgorod, Russia

Institute of Cybernetics, Tallinn, Estonia

Tsunami Wave Shapes at Japanese Coast

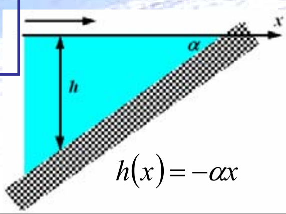


26 May 1983 Japan Sea (Shuto, 1983)

Nonlinear Shallow Water Theory

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}\left[(-\alpha x + \eta)u\right] = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial \eta}{\partial x} = 0$$



$$h(x) = -\alpha x$$

Water waves of finite amplitude on a sloping beach

By G. F. CARRIER and H. P. GREENSPAN
Pierce Hall, Harvard University

(*Received* 2 *December* 1957)

SUMMARY

In this paper, we investigate the behaviour of a wave as it climbs a sloping beach. Explicit solutions of the equations of the non-linear inviscid shallow-water theory are obtained for several physically interesting wave-forms. In particular it is shown that waves can climb a sloping beach without breaking. Formulae for the motions of the instantaneous shoreline as well as the time histories of specific wave-forms are presented.

Hodograph Transformation

$$\frac{\partial^2 \Phi}{\partial \lambda^2} - \frac{\partial^2 \Phi}{\partial \sigma^2} - \frac{1}{\sigma}\frac{\partial \Phi}{\partial \sigma} = 0$$

$$\sigma = 2\sqrt{g(h+\eta)} \geq 0$$

Implicit form

$$\eta = \frac{1}{2g}\left(\frac{\partial \Phi}{\partial \lambda} - u^2\right)$$

$$u = \frac{1}{\sigma}\frac{\partial \Phi}{\partial \sigma}$$

$$x = \frac{1}{2\alpha g}\left(\frac{\partial \Phi}{\partial \lambda} - u^2 - \frac{\sigma^2}{2}\right)$$

$$t = \frac{1}{\alpha g}\left(\lambda - \frac{1}{\sigma}\frac{\partial \Phi}{\partial \sigma}\right)$$

Implicit Form – A Few Exact and Approximated Solutions For Whole Field as Function of (x,t)

Papers (after 2000):

Carrier G.F., Wu T.T., Yeh H. Tsunami run-up and draw-down on a plane beach. *J. Fluid Mech.*, 2003, v. 475, 79-99.

Kânoğlu U. Nonlinear evolution and runup-rundown of long waves over a sloping beach. *J. Fluid Mech.*, 2004, v. 513, 363-372.

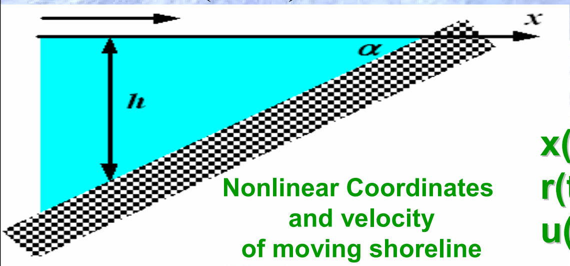
Tinti S., Tonini R. Analytical evolution of tsunamis induced by near-shore earthquakes on a constant-slope ocean. *J. Fluid Mech.*, 2005, v. 535, 33-64.

Kânoğlu U., and Synolakis, C. Initial value problem solution of nonlinear shallow water-wave equations. *Phys. Review Lett.*, 2006, v. 97, 148501.

Explicit Solution for Moving Shoreline if Incident Wave is given Far from Shoreline where It is Linear

Pelinovsky & Mazova, 1992

$$u(t) = U\left(t + \frac{u}{\alpha g}\right) \qquad r(t) = \alpha \int u(t)dt$$



Nonlinear Coordinates and velocity of moving shoreline

x(t)
r(t)
u(t)

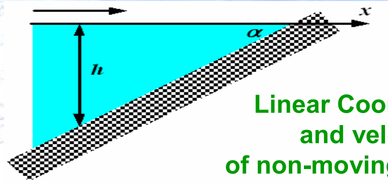
First Step – Solution of Linear Equations For Wave Transformation on Beach

Incident Wave

$$\eta(t) = \sum_{n=1}^{\infty} A_n \sin(n\omega t + \varphi_n)$$

$$R(t) = 2\pi\sqrt{\frac{2L}{\lambda}} \sum_{n=1}^{\infty} \sqrt{n} A_n \sin\left(n\omega t + \varphi_n + \frac{\pi}{4}\right)$$

$$U(t) = \frac{1}{\alpha}\frac{dR}{dt}$$



Linear Coordinates and velocity of non-moving shoreline

x=0
R(t)
U(t)

Second Step – "Nonlinear" Moving Shoreline

$$u(t) = U\left(t + \frac{u}{\alpha g}\right) \qquad r(t) = \alpha \int u(t)dt$$

Linear Coordinates and velocity of non-moving shoreline

x=0, R(t), U(t)

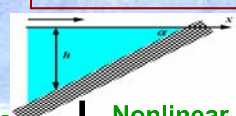
Nonlinear Coordinates and velocity of moving shoreline

x(t), r(t), u(t)

First Result: Linear Theory predicts Maxima

$$u_{\max} = U_{\max}$$
$$r_{\max} = R_{\max}$$

for sine wave $\quad \frac{R}{H_0} = 2\pi\sqrt{\frac{2L}{\lambda_0}}$



Linear Coordinates and velocity of non-moving shoreline

x=0, R(t), U(t)

Nonlinear Coordinates and velocity of moving shoreline

x(t), r(t), u(t)

Natural Hazards **4**: 221–234, 1991.

Tsunami Runup on Steep Slopes: How Good Linear Theory Really Is

COSTAS EMMANUEL SYNOLAKIS
School of Engineering, University of Southern California, Los Angeles, CA 90089-2531, U.S.A.

(Received: 28 January 1990; revised: 29 June 1990)

Abstract. This is a study of the application of linear theory for the estimation of the maximum runup height of long waves on plane beaches. The linear theory is reviewed and a method is presented for calculating the maximum runup. This method involves the calculation of the maximum value of an integral, now known as the runup integral. Laboratory and numerical results are presented to support this method. The implications of the theory are used to reevaluate many existing empirical runup correlations. It is shown that linear theory predicts the maximum runup satisfactorily. This study demonstrates that it is now possible to match complex offshore wave-evolution algorithms with linear theory runup solutions for the purpose of obtaining realistic tsunami inundation estimates.

Second Result: Linear Theory "predicts" Wave Breaking

$$u(t) = U\left(t + \frac{u}{\alpha g}\right)$$

$$\frac{\partial u}{\partial t} = \frac{U'}{1 - \frac{U'}{\alpha g}}$$

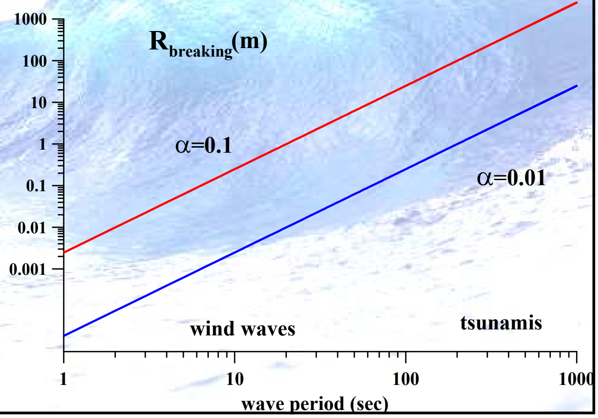
$$Br = \frac{1}{g\alpha^2}\max\left(\frac{d^2R}{dt^2}\right) = 1$$

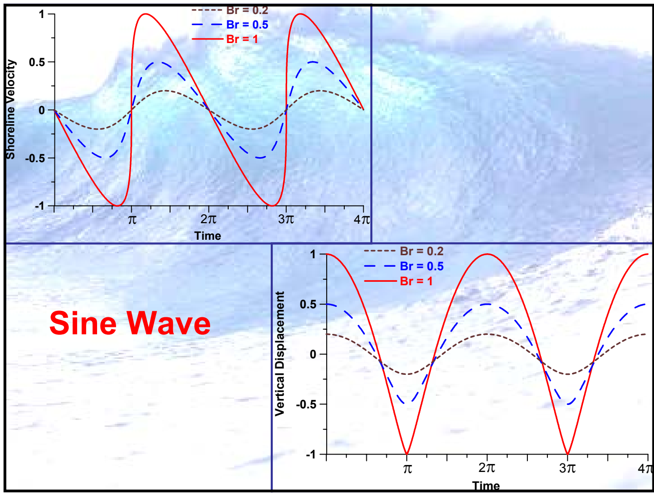
The same as Jacobian Transformation

$$\frac{\partial(t,x)}{\partial(I_+, I_-)} = 0$$

"Linear" Vertical Acceleration = Gravity Acceleration

75% of tsunamis are not breaking waves,
Mazova, Pelinovsky and Soloviev, 1982

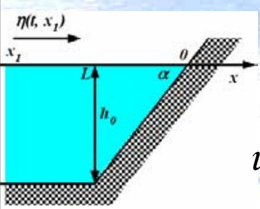




Nonlinear Deformation

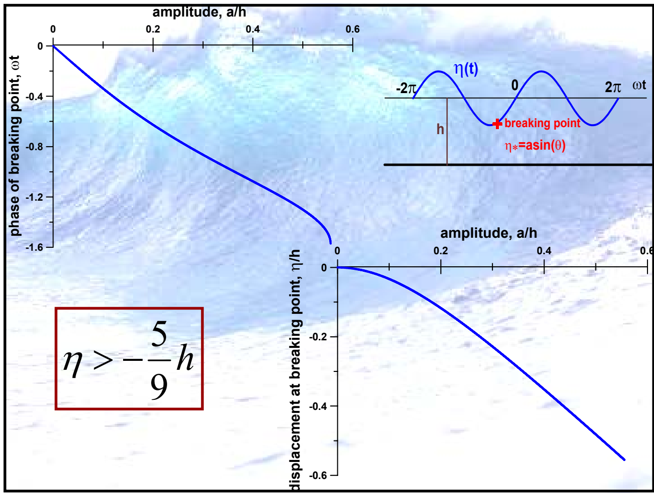
$$\frac{\partial \eta}{\partial t} + V\frac{\partial \eta}{\partial x} = 0$$

$$\eta(x,t) = \eta_0\left(t - \frac{x}{V(\eta)}\right)$$

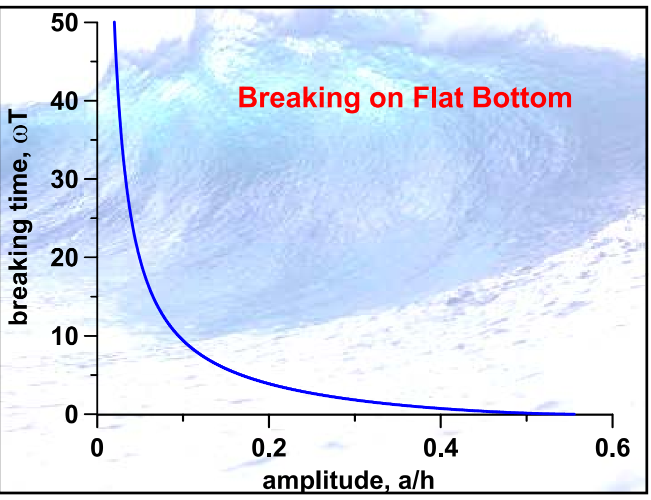


$$V = 3\sqrt{g(h+\eta)} - 2\sqrt{gh}$$

$$u = 2\left(\sqrt{g(h+\eta)} - \sqrt{gh}\right)$$



$$\eta > -\frac{5}{9}h$$



Indian Ocean Estimates

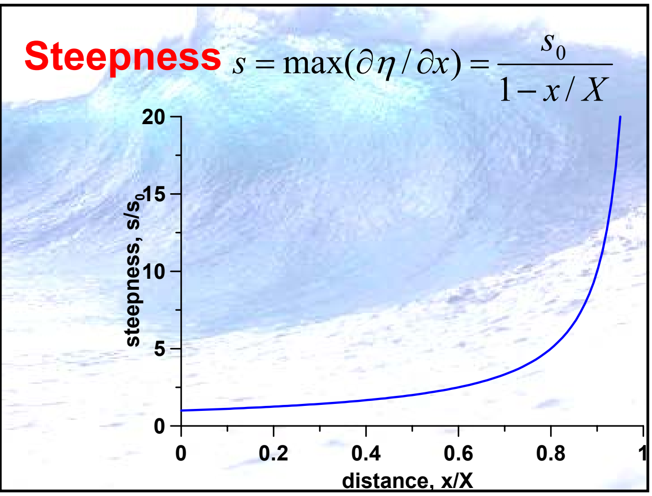
$$X \sim 0.1\lambda\frac{h}{a}$$

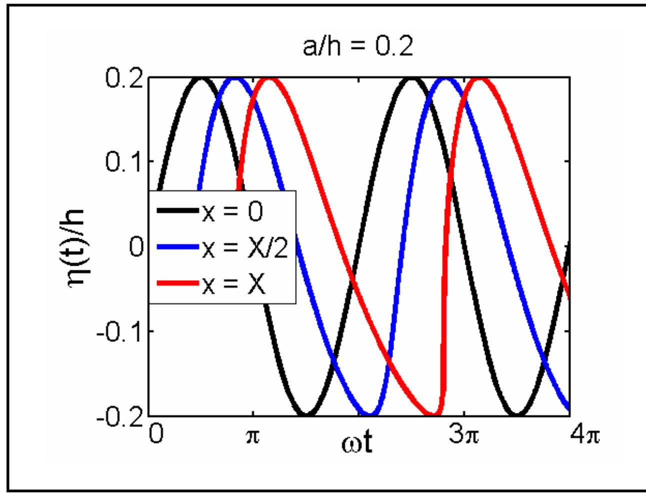
$$X = \frac{2\sqrt{gh}h}{3\omega a} = \frac{\lambda h}{3\pi a}$$

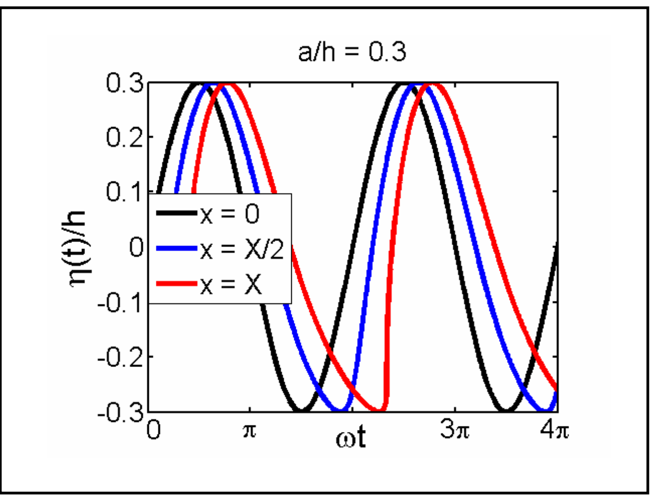
h = 1 km
a = 2 m
λ = 50 km

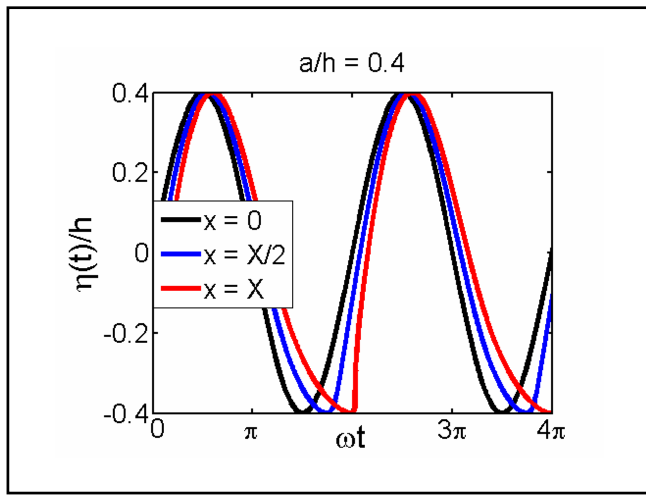
X ~ 2500 km

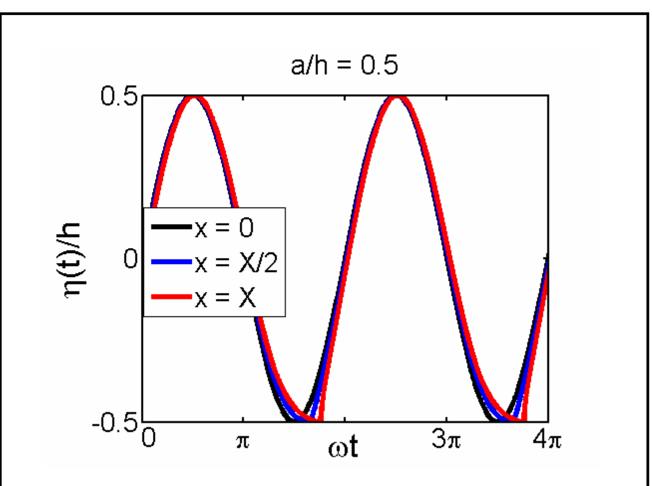
To India!

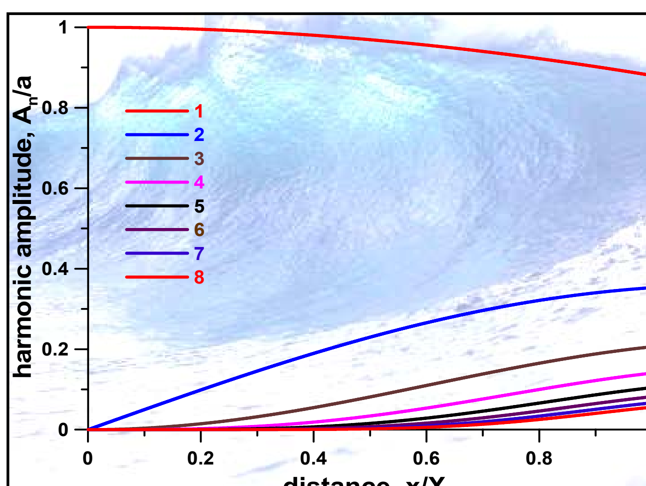

Steepness $s = \max(\partial\eta/\partial x) = \frac{s_0}{1 - x/X}$
steepness, s/s_0
distance, x/X
0
5
10
15
20
0
0.2
0.4
0.6
0.8
1

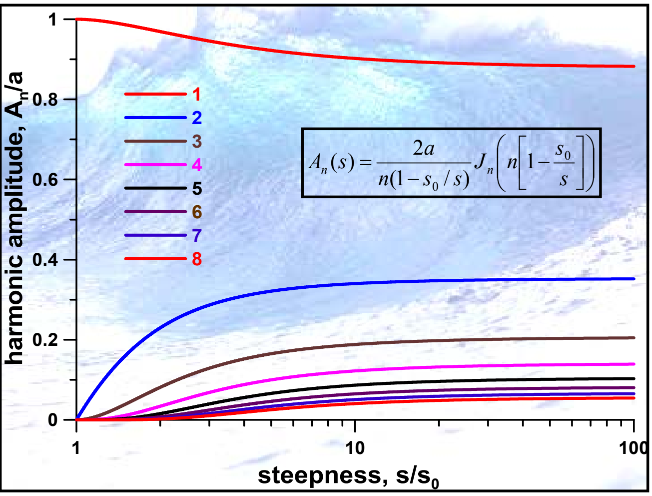

a/h = 0.2
η(t)/h
ωt
x = 0
x = X/2
x = X
0.2
0.1
0
-0.1
-0.2
0
π
3π
4π

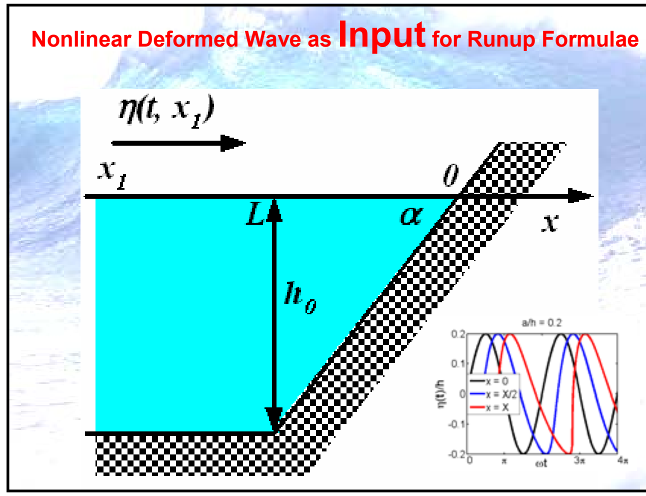

a/h = 0.3
η(t)/h
ωt
x = 0
x = X/2
x = X
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
π
3π
4π

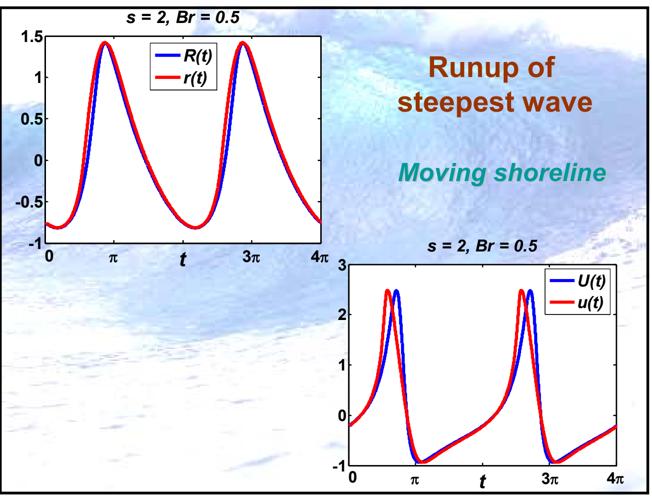

a/h = 0.4
η(t)/h
ωt
x = 0
x = X/2
x = X
0.4
0.2
0
-0.2
-0.4
0
π
3π
4π

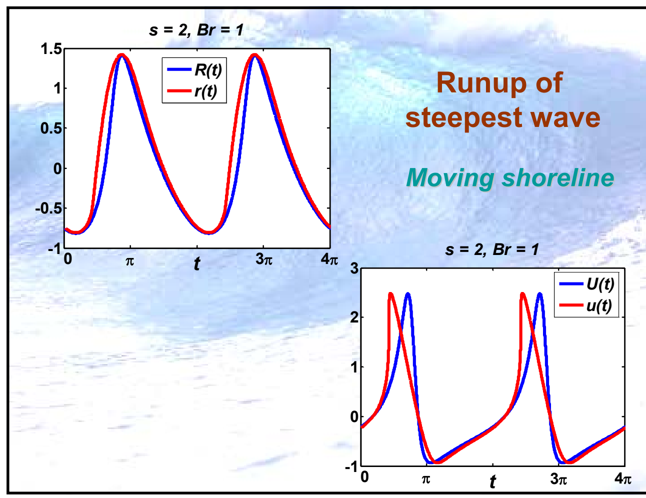

a/h = 0.5
η(t)/h
ωt
x = 0
x = X/2
x = X
0.5
0
-0.5
0
π
3π
4π

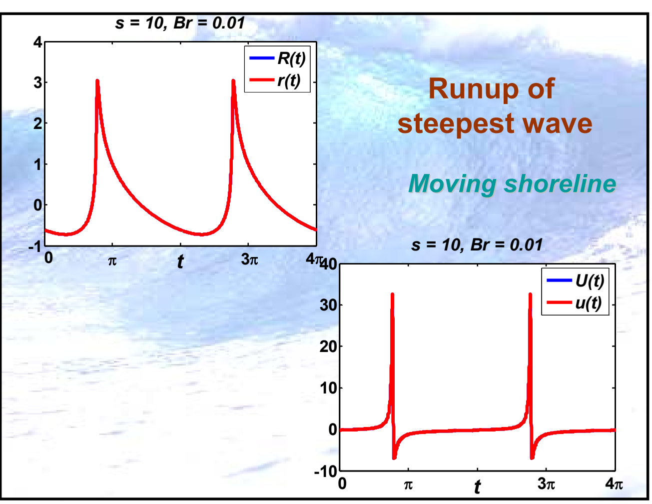

harmonic amplitude, A_n/a
distance, x/X
1
2
3
4
5
6
7
8
0
0.2
0.4
0.6
0.8
1

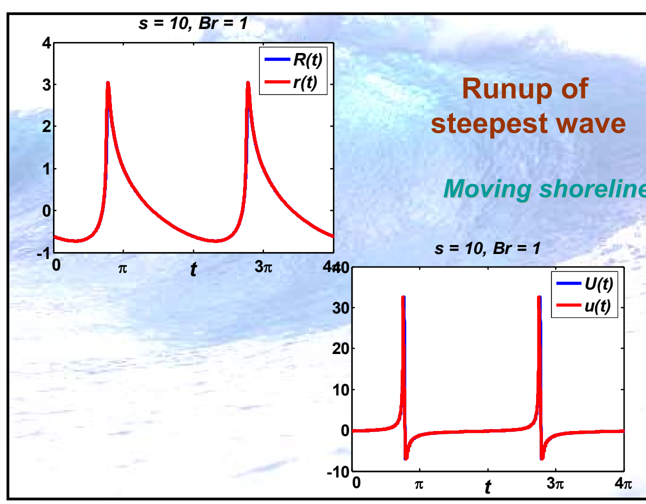

harmonic amplitude, A_n/a
steepness, s/s_0
$A_n(s) = \frac{2a}{n(1-s_0/s)} J_n\left(n\left[1-\frac{s_0}{s}\right]\right)$
1
2
3
4
5
6
7
8

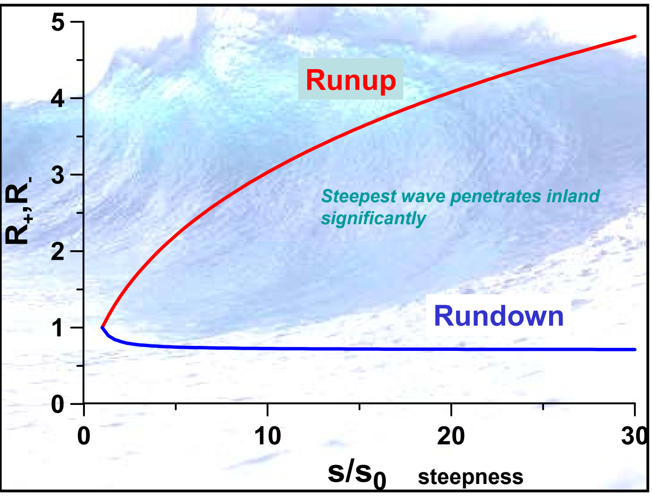

Nonlinear Deformed Wave as Input for Runup Formulae
$\eta(t, x_1)$
x_1
0
x
L
α
h_0

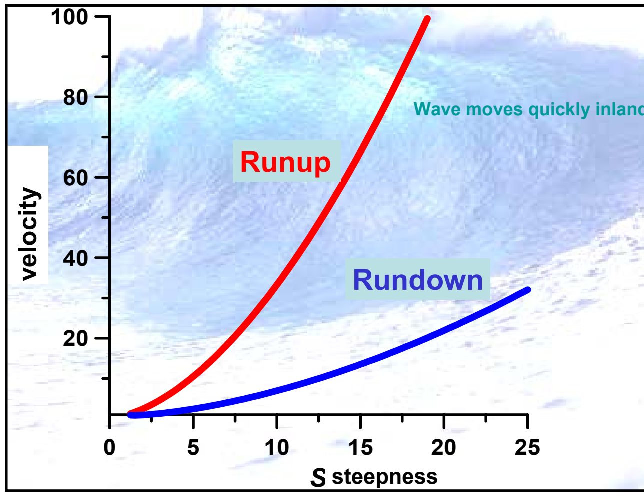

s = 2, Br = 0.5
R(t)
r(t)
Runup of steepest wave
Moving shoreline
s = 2, Br = 0.5
U(t)
u(t)


s = 2, Br = 1
R(t)
r(t)
Runup of steepest wave
Moving shoreline
s = 2, Br = 1
U(t)
u(t)


s = 10, Br = 0.01
R(t)
r(t)
Runup of steepest wave
Moving shoreline
s = 10, Br = 0.01
U(t)
u(t)


s = 10, Br = 1
R(t)
r(t)
Runup of steepest wave
Moving shoreline
s = 10, Br = 1
U(t)
u(t)





ISSN 1028-334X, Doklady Earth Sciences, 2006, Vol. 411, No. 8, pp. 1241–1243. © Pleiades Publishing, Inc., 2006.
Original Russian Text © I.I. Didenkulova, N. Zahibo, A.A. Kurkin, B.V. Levin, E.N. Pelinovsky, T. Soomere, 2006, published in Doklady Akademii Nauk, 2006, Vol. 410, No. 5, pp. 676–678.

GEOPHYSICS

Runup of Nonlinearly Deformed Waves on a Coast

I. I. Didenkulova[a, b], N. Zahibo[c], A. A. Kurkin[b], Corresponding Member of the RAS B. V. Levin[d, e], E. N. Pelinovsky[a, b], and T. Soomere[f]

Received February 2, 2006

ISSN 0001-4338, Izvestiya, Atmospheric and Oceanic Physics, 2006, Vol. 42, No. 6, pp. 773–776. © Pleiades Publishing, Inc., 2006.
Original Russian Text © I.I. Didenkulova, N. Zahibo, A.A. Kurkin, E.N. Pelinovsky, 2006, published in Izvestiya AN. Fizika Atmosfery i Okeana, 2006, Vol. 42, No. 6, pp. 839–842.

Steepness and Spectrum of a Nonlinearly Deformed Wave on Shallow Waters

I. I. Didenkulova[a,b], N. Zahibo[c], A. A. Kurkin[a], and E. N. Pelinovsky[a,b]

Does Runup Height depend on the Incident Wave Shape if the Incident Wave is symmetrical?

For periodic wave – yes!

The anomalous behavior of the runup of cnoidal waves

Costas Emmanuel Synolakis and Manas Kumar Deb
School of Engineering, University of Southern California, Los Angeles, California 90089
James Eric Skjelbreia
MARINETEK, Hakonsongst 34, 7002 Trondheim, Norway

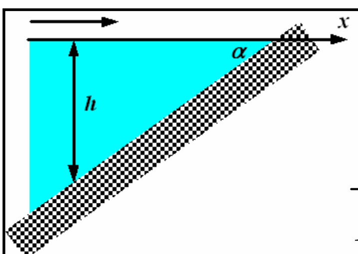
Phys. Fluids, 1988, v. 31, No. 1

For single wave???

Incident wave shapes used:

1. Solitary Wave $f(\mathrm{x}) = \mathrm{sech}^2(\mathrm{x})$
2. Gaussian Pulse $f(x) = \exp(-x^2)$
3. Lorentz Pulse $f(\mathrm{x}) = \dfrac{1}{1+x^2}$

and several others

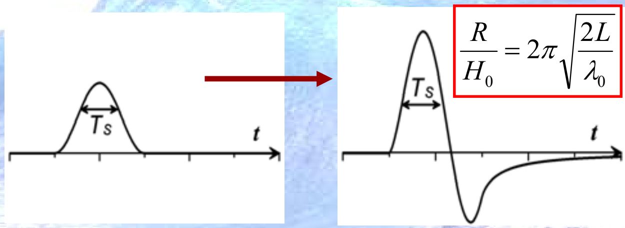


Sine Wave

$$\frac{R}{H_0} = 2\pi\sqrt{\frac{2L}{\lambda_0}}$$

How this formula can be modified?

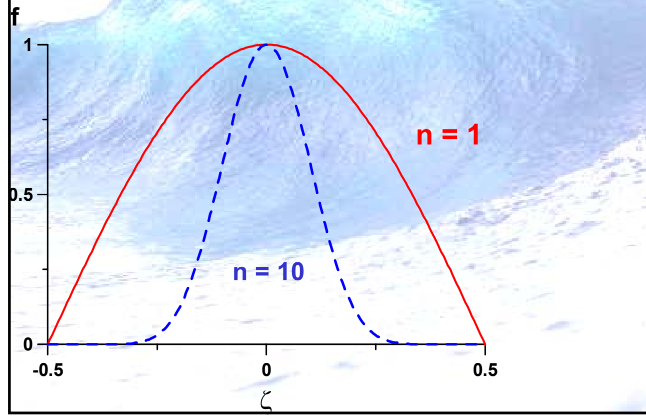
Wave Length (Duration) Definition for Pulse



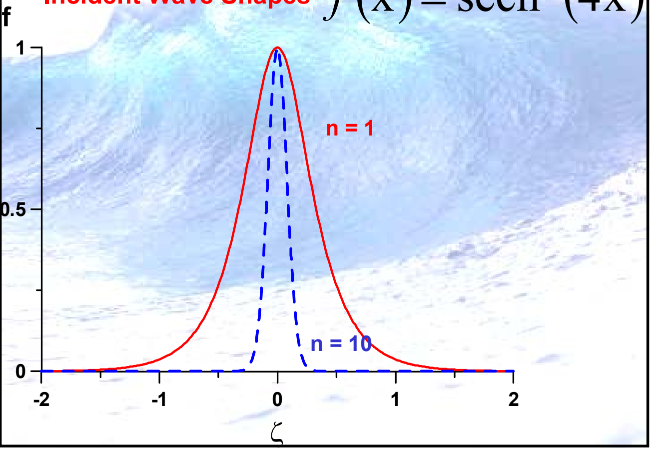
$$\frac{R}{H_0} = 2\pi\sqrt{\frac{2L}{\lambda_0}}$$

We suggest to use length of the wave on 2/3 level – philosophy used to define the significant wave properties

Incident Wave Shapes $f(x) = \cos^n(\pi x)$



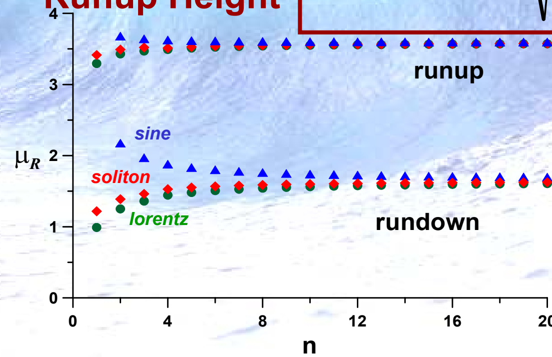
Incident Wave Shapes $f(x) = \mathrm{sech}^n(4x)$



Runup Height

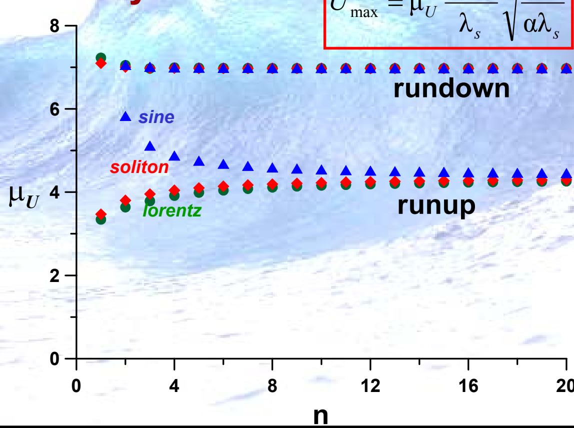
$$R = 2\pi H_0\sqrt{\frac{2L}{\lambda_0}}$$

$$R_{max} = \mu_R^+ H_0\sqrt{\frac{L}{\lambda_s}}$$

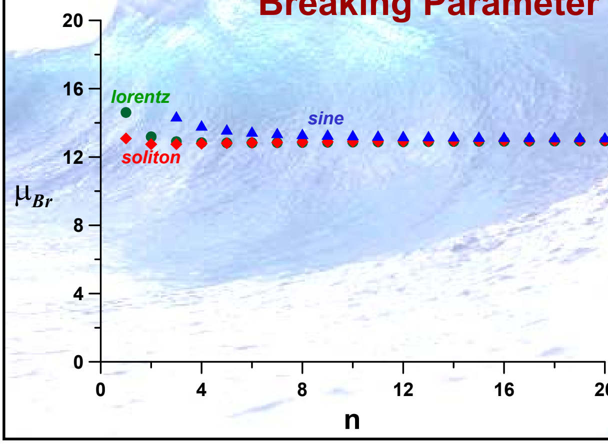


Velocity

$$U_{max} = \mu_U^+ \frac{H_0 L}{\lambda_s}\sqrt{\frac{g}{\alpha\lambda_s}}$$



Breaking Parameter



Parametrized Formulas

$$R_{\text{runup}} = 3.5 H_0 \sqrt{\frac{L}{\lambda_s}}$$

$$R_{\text{down}} = 1.5 H_0 \sqrt{\frac{L}{\lambda_s}}$$

$$U_{runup} = 4.5 \frac{H_0 L}{\lambda_s} \sqrt{\frac{g}{\alpha \lambda_s}}$$

$$U_{rundown} = 7 \frac{H_0 L}{\lambda_s} \sqrt{\frac{g}{\alpha \lambda_s}}$$

$$Br = 13 \frac{H_0 L}{\alpha \lambda_s^{\,2}} \sqrt{\frac{L}{\lambda_s}}$$

Applications:

Easy Estimates of Runup Characteristics

ISSN 0001-4338, Izvestiya, Atmospheric and Oceanic Physics, 2007, Vol. 43, No. 3, pp. 384–390. © Pleiades Publishing, Ltd., 2007.
Original Russian Text © I.I. Didenkulova, A.A. Kurkin, E.N. Pelinovsky, 2007, published in Izvestiya AN. Fizika Atmosfery i Okeana, 2007, Vol. 43, No. 3, pp. 419–425.

Run-up of Solitary Waves on Slopes with Different Profiles

I. I. Didenkulova[a, b], A. A. Kurkin[a], and E. N. Pelinovsky[a, b]
[a] Nizhni Novgorod State Technical University, ul. Minina 24, Nizhni Novgorod, 603950 Russia
e-mail: kurkin@kis.ru
[b] Institute of Applied Physics, Russian Academy of Sciences, ul. Ul'yanova 46, Nizhni Novgorod, 603950 Russia
Received September 7, 2006

Conclusion:

Steepest Wave Penetrates Inland on Large Distance and with Large Velocity - and Slowly into the Sea

Formulas for Solitary Wave Runup can be Parameterized

These results are important for express estimates