Numerical simulation of long ship waves

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Outline of talk

Ship waves in coastal waters

Equations and numerical models

Influence of speed and depth variation on ship generated waves

CENS-CMA project: Ship waves in the Tallinn Bay area

Summary

Wash waves from high speed vessels

HSS “Stena Discovery”

Top Speed:

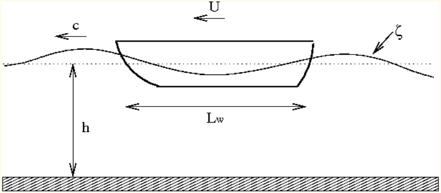
40 knots (= 20.6 m/s)

Dimensions:

Length:	121.75 m
Width:	40.00 m
Draft:	4.80 m

- Waves from high speed vessels:
  - Long wave lengths and wave periods.
  - Large wave energy.
  - Qualitatively different from waves generated by conventional ships.
- Potentially dangerous for people on the shore or in small boats.
- May damage structures at the shore or moored vessels.
- May increase erosion and disturb marine habitats.

Parameters for ship wave generation

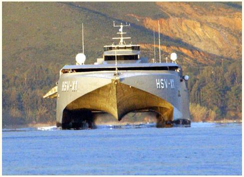


- Length Froude number: $F_L = \frac{U}{\sqrt{gL_w}}$
  - High speed vessels: $F_L > 0.4$
  - Maximum wave resistance (hump speed): $0.4 < F_L < 0.6$
- Depth Froude number: $F_h = \frac{U}{\sqrt{gh}}$
  - Maximum wave resistance: $F_h \approx 1$

Ship classification

Displacement vessels: $F_L < 0.4$
- Hull is supported mainly by buoyancy force

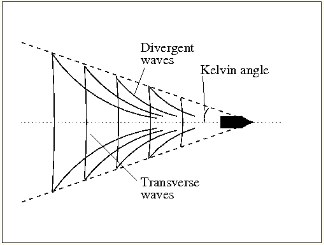
Planing vessels: $F_L > 1.0 - 1.2$
- Hull is supported mainly by hydrodynamic pressure

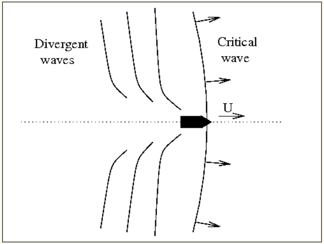


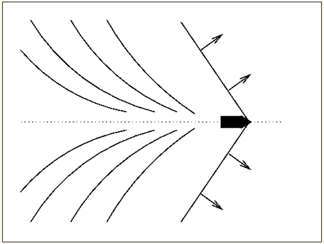
Semi-displacement vessels:
$0.4 - 0.5 < F_L < 1.0 - 1.2$

Ship wave patterns

- Subcritical: $F_h < 0.6$
- Critical: $F_h \approx 1$
- Supercritical: $F_h > 1.2 - 1.4$



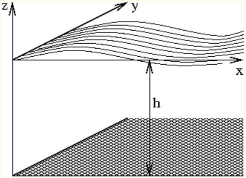




Equations for inviscid, incompressible flow

- Continuity equation:

$$\nabla \cdot \mathbf{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$



- Euler equation:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p + \mathbf{g}$$

- Dynamic and kinematic boundary conditions:

$$\begin{aligned} p &= p_a(x, y, t), \quad &\text{at} \quad z = \eta(x, y, t) \\ w &= \frac{\partial \eta}{\partial t} + u\frac{\partial \eta}{\partial x} + v\frac{\partial \eta}{\partial y}, \quad &\text{at} \quad z = \eta(x, y, t) \\ w &= u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y}, \quad &\text{at} \quad z = -h(x, y) \end{aligned}$$

Strategies for numerical simulations

Numerical solution of Euler equations is possible, but at high computational cost.
- Restrict computational domain (or resolution)

Shallow water approximation:
- Reduce 3D equations to 2D equations by integrating over depth.
- Derive governing equations by expanding **u** in terms of z.

Several different formulations may be derived from the primitive equations.

fKdV and KP equations

Forced KdV (fKdV) equation (1D propagation)

$$\frac{\partial \eta}{\partial t} + \left(1 + \frac{3}{2}\eta\right) \frac{\partial \eta}{\partial t} + \frac{1}{6}\frac{\partial^3 \eta}{\partial x^3} = \frac{1}{2}\frac{\partial p_a}{\partial x}$$

Kadomtsev-Petviashvili (KP) equation (2D propagation)

$$\frac{\partial}{\partial x}\left[\frac{\partial \eta}{\partial t} + \left(1 + \frac{3}{2}\eta\right) \frac{\partial \eta}{\partial t} + \frac{1}{6}\frac{\partial^3 \eta}{\partial x^3}\right] - \frac{1}{2}\frac{\partial^2 \eta}{\partial y^2} = \frac{1}{2}\frac{\partial^2 p_a}{\partial x^2}$$

Derived from KdV equation by relaxing 1D requirement.

Boussinesq equations

- Formulate equations in terms of depth averaged velocity $\bar{\mathbf{u}}$, where $\nabla_H = (\partial/\partial x, \partial/\partial y)$.
- Continuity equation:

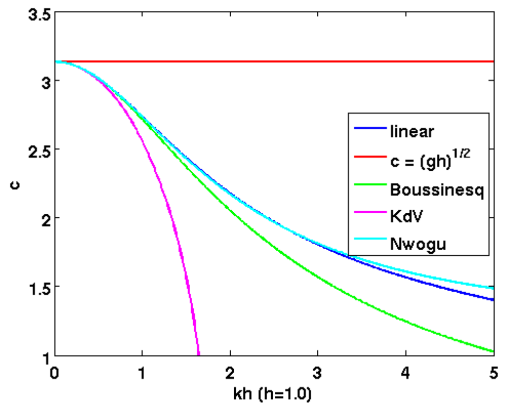
$$\frac{\partial \eta}{\partial t} + \nabla_H \cdot [(h+\eta)\bar{\mathbf{u}}] = 0$$

- Momentum equation:

$$\frac{\partial \bar{\mathbf{u}}}{\partial t} + (\bar{\mathbf{u}} \cdot \nabla_H)\bar{\mathbf{u}} = -\nabla_H \eta - \nabla_H p_a + \frac{1}{3} h^2 \nabla_H \nabla_H \cdot \left(\frac{\partial \bar{\mathbf{u}}}{\partial t}\right)$$

- Extensions of Boussinesq equations:
  - Formulations with depth variation include $\nabla_H h$ terms.
  - Higher order formulations include dispersive terms with nonlinear corrections.

Dispersion relation for long wave equations



Linear: $c = \sqrt{(g/k)\tanh(kh)}$
Boussinesq: Classical Boussinesq formulation
Nwogu: Improved Boussinesq formulation

Properties of long wave equations

KdV and KP equations:

- \+ Closed form (explicit) solutions exist for solitary and periodic waves.
- \- Restricted to unidirectional (KdV) or narrow angle (KP) of wave propagation.
- \- Poor dispersion relation for intermediate water depth.

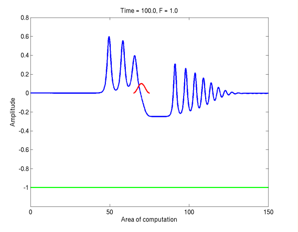
Boussinesq equations:

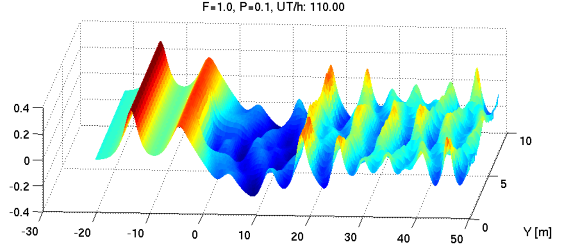
- \- No known closed form solutions.
- \+ No preferred direction of wave propagation.
- \+ Improved formulation with reasonable dispersion relation up to $kh \approx \pi$.

Waves generated by a pressure disturbance

Waves generated by a moving pressure disturbance.

- Simple to implement.
- Unrealistic representation of wave field near the vessel.





Ship waves in coastal waters

Equations and numerical models

Influence of speed and depth variation on ship generated waves

CENS-CMA project: Ship waves in the Tallinn Bay area

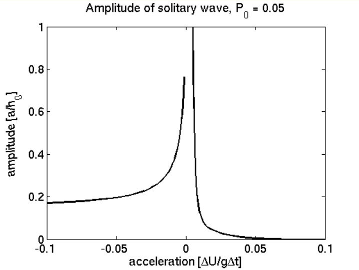
Summary

Variable depth Froude number

- Motivation:
  Study waves generated during the transition between sub- and supercritical speed regimes.
- Method:
  Solve standard Boussinesq equations in one horizontal dimension (1-HD).
  - Weakly nonlinear equations: $\epsilon = \mathcal{O}(\mu^2)$
  - Discretized with finite differences, using a staggered grid in time and space.
  - F_h ($= U/\sqrt{gh}$) changes due to linear variation in U or h.

Variable depth Froude number

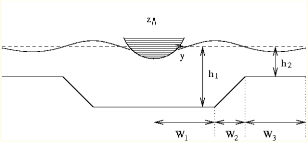
- Results:
  Large amplitude solitary waves may be generated during a slow transition between sub- and supercritical speed.



- A precursor wave is always generated for a transition from super- to subcritical speed.
- The solitary wave may be trapped by the wave generating disturbance during a slow transition from sub- to supercritical speed.

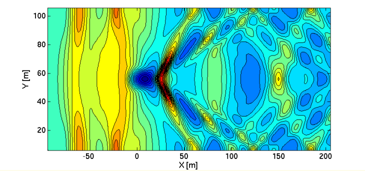
Channels with a deep centre-line trench

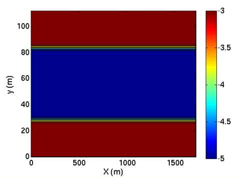
- Motivation: Study waves generated by a vessel in a channel with a deep trench along the centre line.



- Method:
  Simulate waves using the Cornell University Long Wave model (COULWAVE).
  - 2-HD simulations.
  - Fully nonlinear equations: $\epsilon = \mathcal{O}(1)$
  - Equations with improved dispersive properties.
  - Discretized with finite differences.
  - 4th order predictor-corrector method for time integration.

Channels with a deep centre-line trench



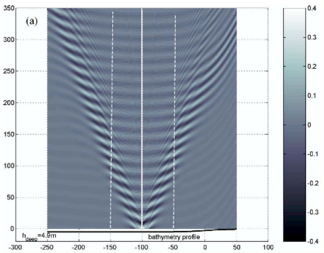


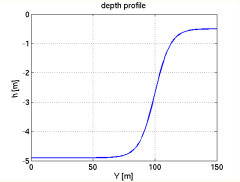
- Wave patterns with near critical or supercritical characteristics may occur, even if the vessel maintains a subcritical speed relative to the depth in the trench.
- When the leading precursor wave is long relative to the channel width, it can be described by cross-channel averaged theory.
- Large amplitude waves may sometimes occur on the shallow banks near the channel walls due to wave interactions in the downstream wave pattern.

Refraction of waves over a sloping bottom profile

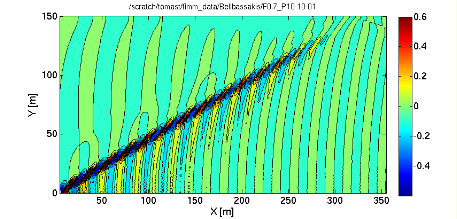
Results for $F_h = 0.7$

Belibassakis (2003)

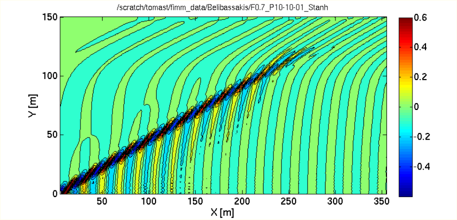




Constant depth



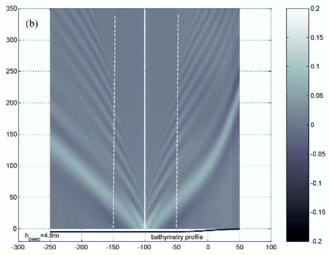
Sloping bottom profile

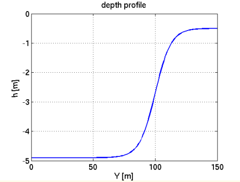


Refraction of waves over a sloping bottom profile

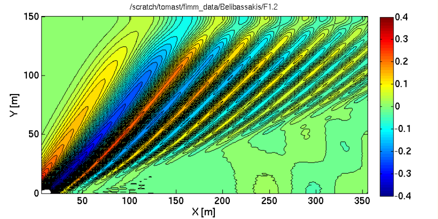
Results for $F_h = 1.2$

Belibassakis (2003)

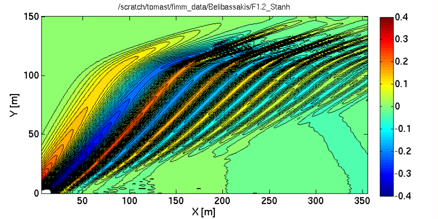




Constant depth



Sloping bottom profile



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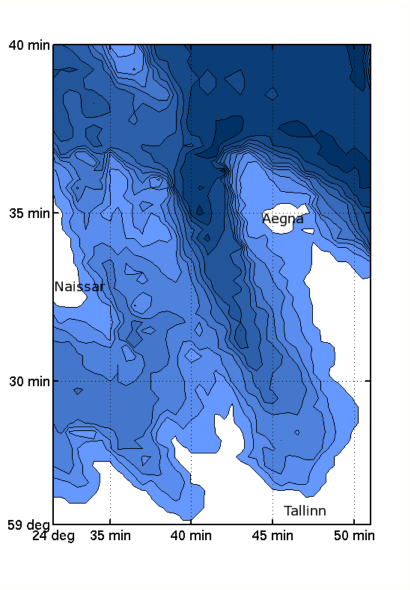
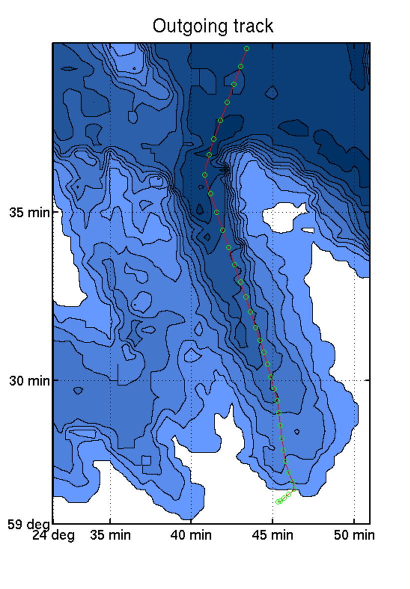
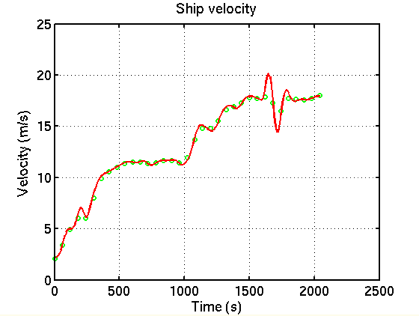
Ship waves in the Tallinn Bay area



- Basin of about 16 km x 16 km
- Ship traffic follows NW - SE underwater valley, with depth ranging from 10 m to 90 m.
- Daily crossings (ingoing or outgoing)
  - 22 HSC/Catamaran
  - 8-10 Hydrofoil
- A natural laboratory for the study of long waves.
  - Also relevant for tsunami research.

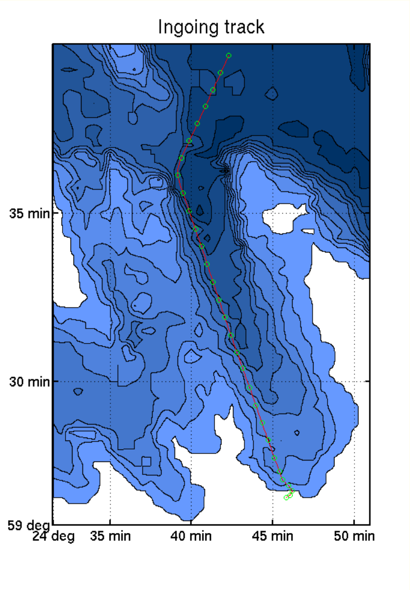
Ship track for outgoing ship

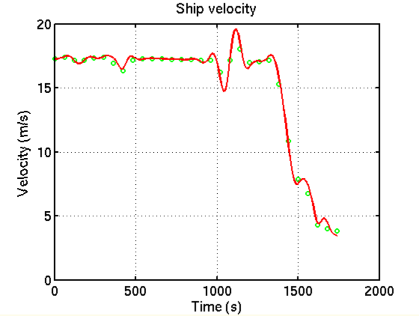




- $U \approx 12$ m/s in inner part of the bay.
- $U \approx 17$ m/s as the ship approaches Aegna.
- Turn outside Aegna may cause wave focusing.

Ship track for ingoing ship





- $U \approx 17$ maintained almost the entire length of the bay.
- Ship track coincides with lateral slope of the underwater valley.

Summary

- Long waves generated by high speed vessels are
  - a recent addition to the typical wave spectre in coastal areas.
  - a potential safety and environmental hazard.
- The long wave part of the ship wash is well described by the Boussinesq equations.
- For practical applications, particular attention should be placed on effects due to ship acceleration and maneuvering, as well as effects due to a variable bathymetry.